national accelerator laboratory

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ACCELERATOR EXPERIMENT: Vertical Incoherent Tune Shift in the Main Ring Due to Space Charge Effects

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The tune of the incoherent betatron oscillation is expected to be shifted by the space charge of the beam itself and by the image charge created in the surrounding material (vacuum pipe and magnets). Traditionally, this shift is used to set the so-called space-charge limit of accelerators.[1] The measurement of the incoherent tune shift cannot be done by pinging the beam since the tune shift in that case will be the coherent tune shift.[2]

The measurement reported here utilizes the beam loss at the third-integer resonance, either $3\nu_x = 58$ or $3\nu_y = 58$. If the intensity of the beam is changed, the tune will be shifted by the amount proportional to the change in intensity and the beam loss due to the third-integer resonance will be observed at a different quadrupole setting. Since the force due to space charge is repulsive, the tune will be shifted down when the intensity is increased and the resonance will be observed at a higher quadrupole strength. This is shown in Figs. 1 and 2. The abscissa should be regarded as an indication of quadrupole setting so that only the difference in tune is meaningful. The shift of the vertical tune is 0.01 when the intensity is increased from $5\text{x}10^{11}$ to $1\text{x}10^{12}$. Since the tune shift is proportional to the intensity, one can conclude that $\Delta\nu_y$ (incoherent) = -0.02 for $1\text{x}10^{12}$ protons at 8 GeV. In this experiment, only one booster pulse (with 83 RF bunches) was injected into the main ring and the resonance was located as a dip in the transmission at 0.3 sec. The intensity was reduced by deliberately detuning quadrupoles in the 200 MeV line. This tends to saturate the aperture of the booster and the transverse beam size is not affected by the reduction in intensity. The beam size at β_{max} was ±1 cm in both directions. There was no detectable tune shift in the horizontal direction under the same condition. However, the transmission at the dip was only 20% and points were scattered in a wide range. Ideally speaking, the intensity should not change during a measurement - an impossible requirement for the method used here.

One can calculate the expected tune shift from the classical Laslett formula.[3] This assumes a uniformly charged elliptic cylinder and the image force is calculated for a line source in the middle of two parallel conducting planes and two parallel magnetic pole surfaces. For 1×10^{12} protons in 83 RF bunches, one gets[4]

$$\Delta\nu_y \text{ (incoherent)} = -0.0404$$

of which -0.0360 is due to the self field and -0.0044 from the image force. This is twice the observed value. If one simply extends the arithmetic to 13 booster pulses, each with 4×10^{12} protons (total 5.2×10^{13} protons), $\Delta\nu_y = -0.30$ which may mean $\Delta\nu_y = -0.15$ if the discrepancy between the calculation and the measurement is blindly applied.

Following arguments have been contributed, individually and collectively, by Tom Collins, Sandro Ruggiero and Lee Teng in order to "explain" the discrepancy.

(1) The beam size may have increased when the intensity was doubled. If max. $x,y = \pm 1.1$ cm for 1×10^{12} protons and ± 1 cm for 5×10^{11} particles, the change in the shift becomes 0.014 instead of 0.02.

(2) The intensity drops to 60% at the dip of the transmission. This would suggest that the intensity to be used in the formula should be ~80% ($.8 \times 10^{12}$ instead of 1×10^{12}).

(3) The charge distribution must be closer to Gaussian than uniform. The tune shift is then <u>larger</u> for a small amplitude particle but <u>smaller</u> for a large amplitude particle. Since one is observing the loss, the tune involved would be that of large amplitude.

(4) Since the beam size is not much smaller than the vertical gap, a small amount of <u>coherent</u> vertical oscillation will change the effect of image charge.

(5) Laslett formula assumes that all <u>ac</u> components of the magnetic field will be stopped at the vacuum pipe wall. The

thickness of the main ring vacuum pipe (.05", stainless steel) corresponds to the skin depth of 125 kHz. Since the revolution frequency is 47 kHz, part of ac components will go through the pipe. This tends to increase the tune shift compared to the value from Laslett formula.

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1. See, for example, NAL Design Report ("White Book"), p. 4-16
2. Accelerator Experiment EXP-65
3. L.J. Laslett, BNL-7534, July 1963, p. 324, CERN "Yellow" Booklet (CERN/MPS-SI/Int. DL/70/4). p. 26
4. Following parameters were used.

   Bunching factor B = (83x0.6 m)/(2πx1,000 m) = .00793

   total number $N = 1\text{x}10^{12}$

   transverse emittance = $\pi\text{x}10^{-6}$ m (both directions)

   $\gamma = 9.53$

   Laslett parameters $\varepsilon_1 = 0.2$, $\varepsilon_2 = 0.4$

   proton classical radius $r_p = 1.535\text{x}10^{-18}$ m

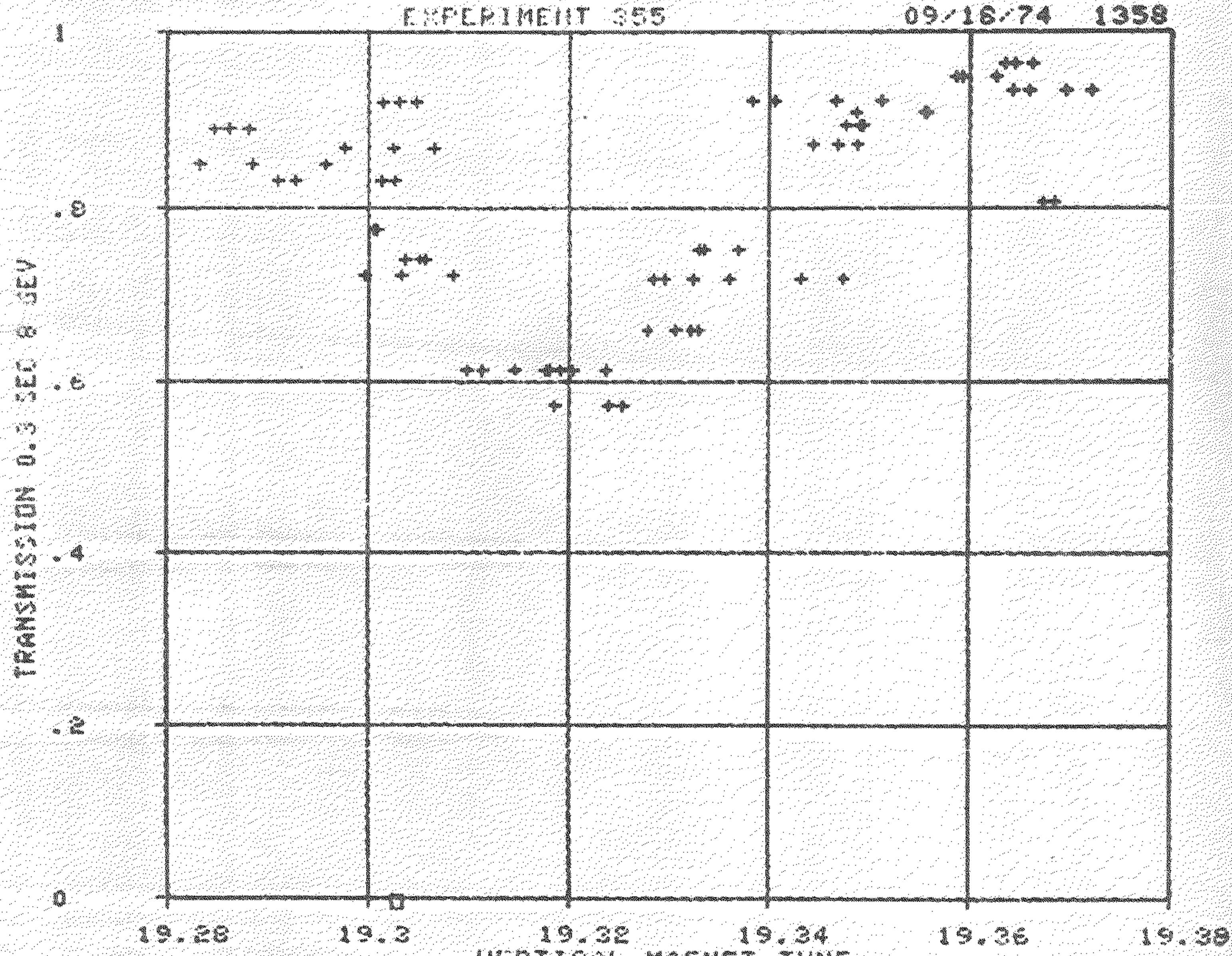
   conductor gap (half size) h = 1.75 cm (B1)
   = 2.41 cm (B2 and quads)

   magnet pole gap (half size) g = 1.91 cm (B1)
   = 2.5 cm (B2 and quads)

   average β_y = 39 m (B1)
   = 72 m (B2)
   = 65 m (quads)

   Image effects are ignored in the drift space.

Fig. 1 5x10" protons in 83 RF bunches.
8GeV
max. x, y = ±1cm.
each bunch ~60cm long



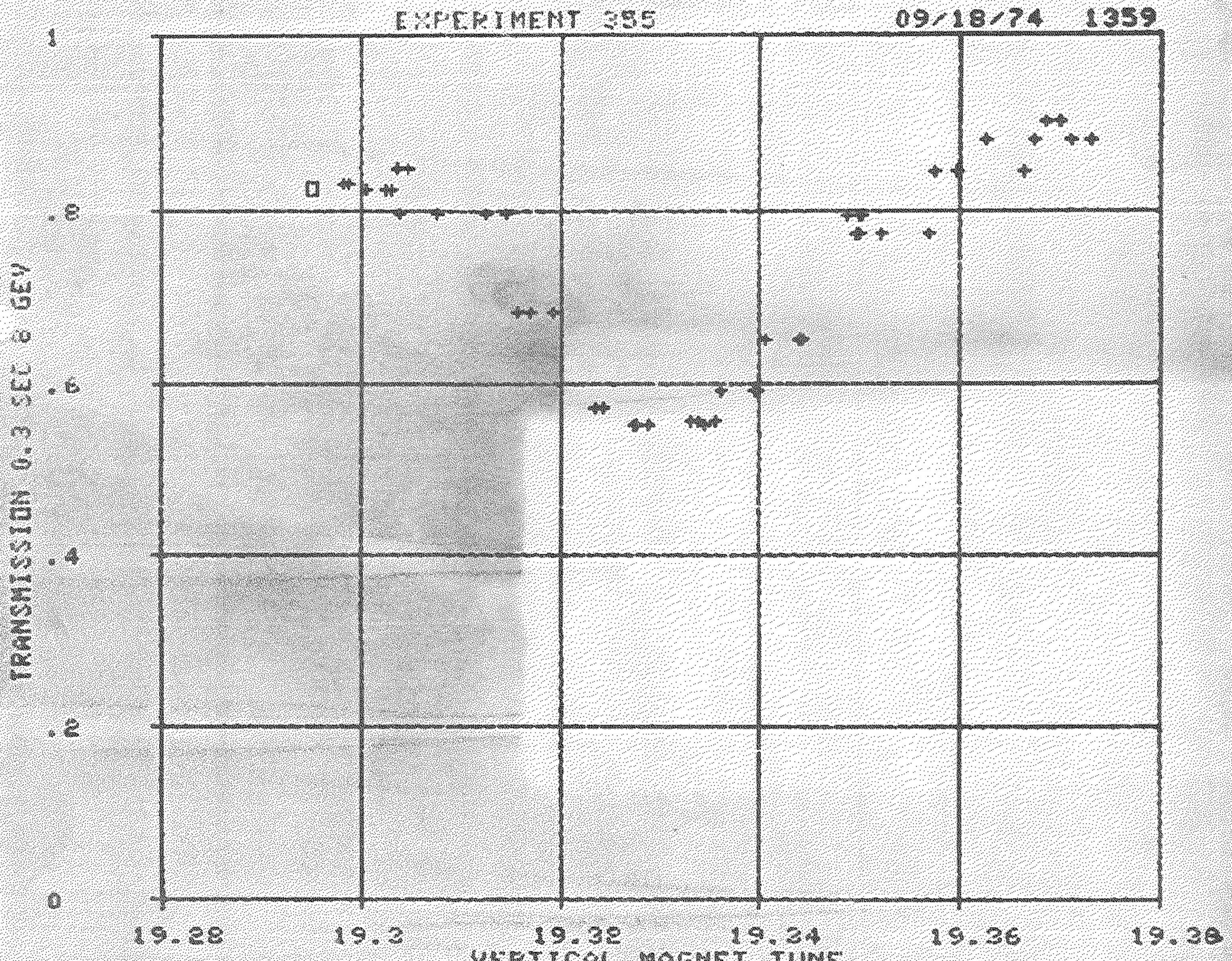
(quads setting)

Fig. 2 1×10^{12} protons in 83 RF bunches.

8 GeV

max. $x, y = \pm 1$ cm

each bunch ~ 60 cm long.



(quads setting)